

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* \mathbf{M} of the force. Moment is also referred to as *torque*.

As a familiar example of the concept of moment, consider the pipe wrench of Fig. 2/8a. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

Moment about a Point

Figure 2/8b shows a two-dimensional body acted on by a force \mathbf{F} in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis $O-O$ perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm* d , which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

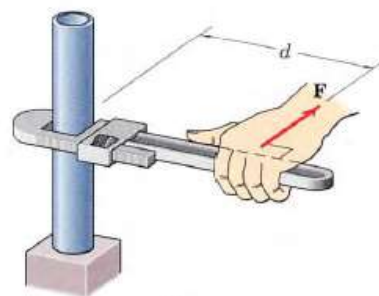
$$M = Fd \quad (2/5)$$

The moment is a vector \mathbf{M} perpendicular to the plane of the body. The sense of \mathbf{M} depends on the direction in which \mathbf{F} tends to rotate the body. The right-hand rule, Fig. 2/8c, is used to identify this sense. We represent the moment of \mathbf{F} about $O-O$ as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency.

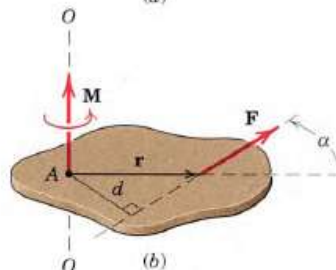
The moment \mathbf{M} obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are newton-meters ($\text{N}\cdot\text{m}$), and in the U.S. customary system are pound-feet ($\text{lb}\cdot\text{ft}$).

When dealing with forces which all act in a given plane, we customarily speak of the moment *about a point*. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force \mathbf{F} about point A in Fig. 2/8d has the magnitude $M = Fd$ and is counterclockwise.

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (−) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig. 2/8d, the moment of \mathbf{F} about point A (or about the z -axis passing through point A) is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.



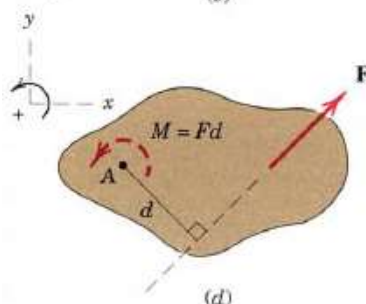
(a)



(b)



(c)



(d)

Figure 2/8

The Cross Product

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. The moment of \mathbf{F} about point A of Fig. 2/8b may be represented by the cross-product expression

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \quad (2/6)$$

where \mathbf{r} is a position vector which runs from the moment reference point A to any point on the line of action of \mathbf{F} . The magnitude of this expression is given by*

$$M = Fr \sin \alpha = Fd \quad (2/7)$$

(b)
Fig. 4-2

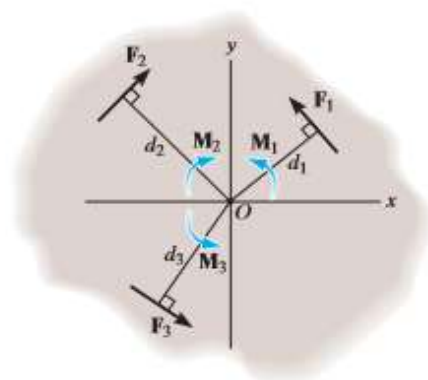


Fig. 4-3

Direction. The direction of M_O is defined by its *moment axis*, which is perpendicular to the plane that contains the force F and its moment arm d . The right-hand rule is used to establish the sense of direction of M_O . According to this rule, the natural curl of the fingers of the right hand, as they are drawn towards the palm, represent the rotation, or if no movement is possible, there is a tendency for rotation caused by the moment. As this action is performed, the thumb of the right hand will give the directional sense of M_O , Fig. 4-2a. Notice that the moment vector is represented three-dimensionally by a curl around an arrow. In two dimensions this vector is represented only by the curl as in Fig. 4-2b. Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page.

Resultant Moment. For two-dimensional problems, where all the forces lie within the x - y plane, Fig. 4-3, the resultant moment $(M_R)_O$ about point O (the z axis) can be determined by finding the algebraic sum of the moments caused by all the forces in the system. As a convention, we will generally consider *positive moments* as *counterclockwise* since they are directed along the positive z axis (out of the page). *Clockwise moments* will be *negative*. Doing this, the directional sense of each moment can be represented by a *plus or minus* sign. Using this sign convention, the resultant moment in Fig. 4-3 is therefore

$$\zeta + (M_R)_O = \sum Fd; \quad (M_R)_O = F_1d_1 - F_2d_2 + F_3d_3$$

If the numerical result of this sum is a positive scalar, $(M_R)_O$ will be a counterclockwise moment (out of the page); and if the result is negative, $(M_R)_O$ will be a clockwise moment (into the page).

Example: For each case illustrated in Fig. 4-4, determine the moment of the force about point O .

SOLUTION (SCALAR ANALYSIS)

The line of action of each force is extended as a dashed line in order to establish the moment arm d . Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about O is shown as a colored curl. Thus,

Fig. 4-4a $M_O = (100 \text{ N})(2 \text{ m}) = 200 \text{ N} \cdot \text{m} \curvearrowright$

Ans.

Fig. 4-4b $M_O = (50 \text{ N})(0.75 \text{ m}) = 37.5 \text{ N} \cdot \text{m} \curvearrowright$

Ans.

Fig. 4-4c $M_O = (40 \text{ lb})(4 \text{ ft} + 2 \cos 30^\circ \text{ ft}) = 229 \text{ lb} \cdot \text{ft} \curvearrowright$

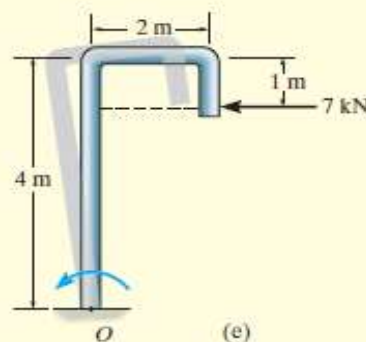
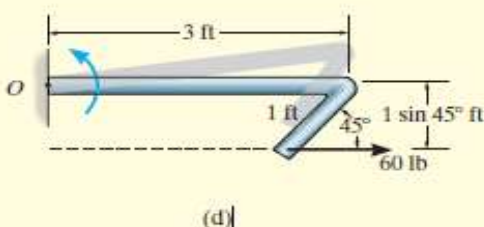
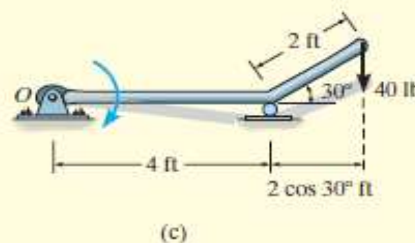
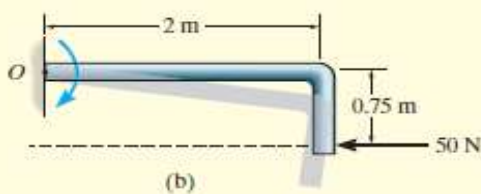
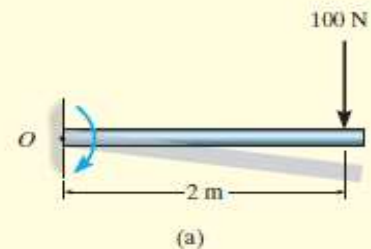
Ans.

Fig. 4-4d $M_O = (60 \text{ lb})(1 \sin 45^\circ \text{ ft}) = 42.4 \text{ lb} \cdot \text{ft} \curvearrowright$

Ans.

Fig. 4-4e $M_O = (7 \text{ kN})(4 \text{ m} - 1 \text{ m}) = 21.0 \text{ kN} \cdot \text{m} \curvearrowright$

Ans.



EXAMPLE 4.2

Determine the resultant moment of the four forces acting on the rod shown in Fig. 4–5 about point O .

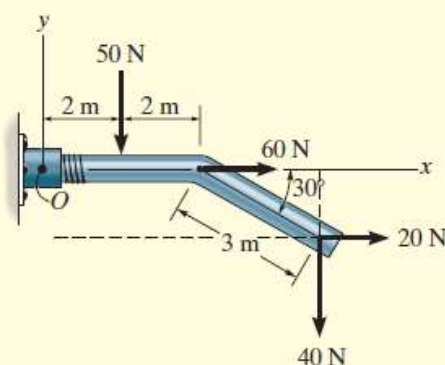


Fig. 4–5

SOLUTION

Assuming that positive moments act in the $+k$ direction, i.e., counterclockwise, we have

$$\zeta + (M_R)_O = \Sigma Fd;$$

$$(M_R)_O = -50 \text{ N}(2 \text{ m}) + 60 \text{ N}(0) + 20 \text{ N}(3 \sin 30^\circ \text{ m})$$

$$-40 \text{ N}(4 \text{ m} + 3 \cos 30^\circ \text{ m})$$

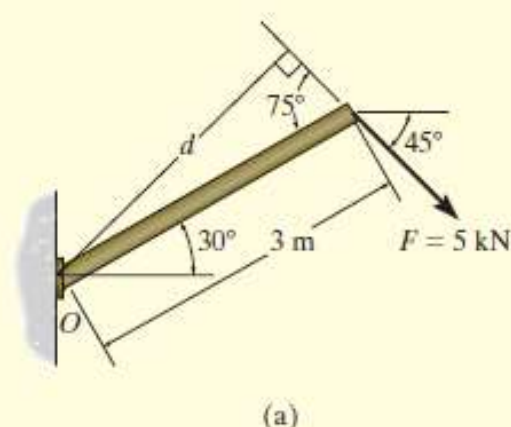
$$(M_R)_O = -334 \text{ N} \cdot \text{m} = 334 \text{ N} \cdot \text{m} \curvearrowright$$

Ans.

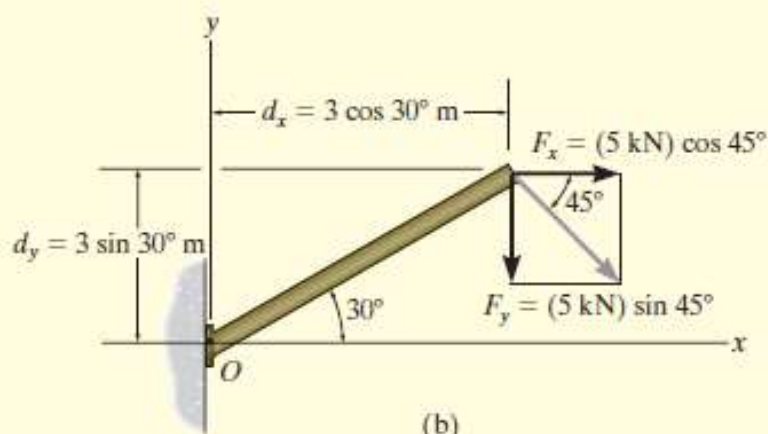
For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.

EXAMPLE 4.5

Determine the moment of the force in Fig. 4–18a about point O .



(a)



(b)

SOLUTION I

The moment arm d in Fig. 4–18a can be found from trigonometry.

$$d = (3 \text{ m}) \sin 75^\circ = 2.898 \text{ m}$$

Thus,

$$M_O = Fd = (5 \text{ kN})(2.898 \text{ m}) = 14.5 \text{ kN} \cdot \text{m} \curvearrowright \quad \text{Ans.}$$

Since the force tends to rotate or orbit clockwise about point O , the moment is directed into the page.

SOLUTION II

The x and y components of the force are indicated in Fig. 4–18b. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\zeta + M_O = -F_x d_y - F_y d_x$$

$$= -(5 \cos 45^\circ \text{ kN})(3 \sin 30^\circ \text{ m}) - (5 \sin 45^\circ \text{ kN})(3 \cos 30^\circ \text{ m})$$

$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \curvearrowright$$

Ans.

SOLUTION III

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4-18c. Here F_x produces no moment about point O since its line of action passes through this point. Therefore,

$$\begin{aligned}\zeta + M_O &= -F_y d_x \\ &= -(5 \sin 75^\circ \text{ kN})(3 \text{ m}) \\ &= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \curvearrowright\end{aligned}$$

Ans.

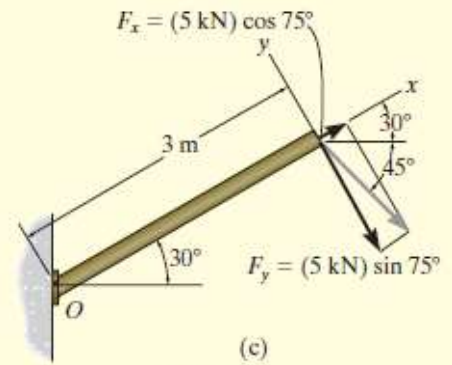
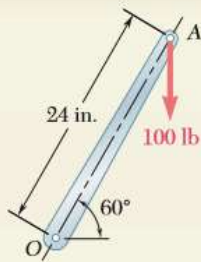
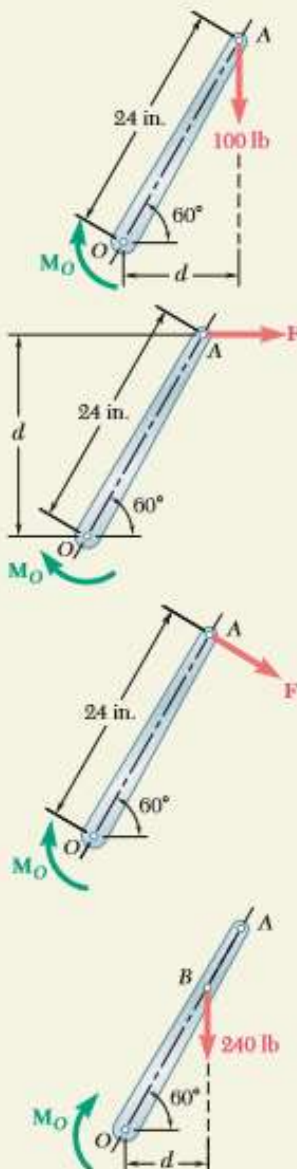


Fig. 4-18



SAMPLE PROBLEM 3.1

A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at O . Determine (a) the moment of the 100-lb force about O ; (b) the horizontal force applied at A which creates the same moment about O ; (c) the smallest force applied at A which creates the same moment about O ; (d) how far from the shaft a 240-lb vertical force must act to create the same moment about O ; (e) whether any one of the forces obtained in parts b , c , and d is equivalent to the original force.



SOLUTION

a. Moment about O . The perpendicular distance from O to the line of action of the 100-lb force is

$$d = (24 \text{ in.}) \cos 60^\circ = 12 \text{ in.}$$

The magnitude of the moment about O of the 100-lb force is

$$M_O = Fd = (100 \text{ lb})(12 \text{ in.}) = 1200 \text{ lb} \cdot \text{in.}$$

Since the force tends to rotate the lever clockwise about O , the moment will be represented by a vector \mathbf{M}_O perpendicular to the plane of the figure and pointing *into* the paper. We express this fact by writing

$$\mathbf{M}_O = 1200 \text{ lb} \cdot \text{in.} \downarrow$$

b. Horizontal Force. In this case, we have

$$d = (24 \text{ in.}) \sin 60^\circ = 20.8 \text{ in.}$$

Since the moment about O must be $1200 \text{ lb} \cdot \text{in.}$, we write

$$\begin{aligned}M_O &= Fd \\ 1200 \text{ lb} \cdot \text{in.} &= F(20.8 \text{ in.}) \\ F &= 57.7 \text{ lb} \quad \mathbf{F} = 57.7 \text{ lb} \rightarrow\end{aligned}$$

c. Smallest Force. Since $M_O = Fd$, the smallest value of F occurs when d is maximum. We choose the force perpendicular to OA and note that $d = 24 \text{ in.}$; thus

$$\begin{aligned}M_O &= Fd \\ 1200 \text{ lb} \cdot \text{in.} &= F(24 \text{ in.}) \\ F &= 50 \text{ lb} \quad \mathbf{F} = 50 \text{ lb} \nearrow 30^\circ\end{aligned}$$

d. 240-lb Vertical Force. In this case $M_O = Fd$ yields

$$\begin{aligned}1200 \text{ lb} \cdot \text{in.} &= (240 \text{ lb})d \quad d = 5 \text{ in.} \\ \text{but} \quad OB \cos 60^\circ &= d \quad \mathbf{OB} = 10 \text{ in.}\end{aligned}$$

e. None of the forces considered in parts b , c , and d is equivalent to the original 100-lb force. Although they have the same moment about O , they have different x and y components. In other words, although each force tends to rotate the shaft in the same manner, each causes the lever to pull on the shaft in a different way.